

## A two DOF spherical parallel platform posture analysis based on kinematic principle

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**ABSTRACT :** *Introducing the principle of kinematic analysis, the mathematical description and analysis method of the constraints, pose and freedom were briefly outlined. A new 2-PSP & 1-S platform configuration was presented. To derive attitude mathematical model of the platform, freedom and spinor system of the two DOF spherical parallel platform were analyzed using kinematics principles. The results show, introducing the concept of position and pose into the kinematic design, kinematic design method can be more widely used to deal with the problem of the movement of the mechanism, so as to expand the application range of kinematic design.*

**Keywords** –Kinematic analysis, Two DOF, Parallel platform, Pose

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### I. INTRODUCTION

Constraint, degree of freedom and position have a fundamental position in kinematics, the kinematics of the trajectory, velocity, acceleration, etc., are their natural mathematical description or derivation. Therefore, kinematics can be attributed to the processing of these three concepts. From a more general point of view, kinematics is geometry because the constraint, degree of freedom and pose of the concept describes the geometric structure of the object motion. As is known to all, institutional innovation is an important aspect of mechanical system innovation. At present, there are many researches on the parallel mechanism of three degrees of freedom at home and abroad, and the research results have been obtained<sup>[1-6]</sup>. But there are two kinds of configuration of the spherical structure of the two degree of freedom: One is the 2-SPS&1-U two DOF spherical parallel platform mechanism proposed by Carricato<sup>[7]</sup>. This mechanism is connected through the two SPS kinematic chain and a U kinematic pair between the moving platform and the base of the mechanism. The intersection of the rotational axis of U Joints is the center of the fixed movement of the moving platform; The other is the kinematic decoupling of the two degree of freedom spherical parallel mechanism proposed by Li Weimin from Hebei University of Technology<sup>[8]</sup>. Each branch of the mechanism of the body is driven by a separate drive that causes the moving platform to rotate around the axis of the shaft and is independent of the other branch of the driving unit, so as to realize the motion decoupling.

In this paper, the concept of pose is introduced into kinematic design, the three kinematic concepts of constraint, degree of freedom and pose were detailedly discussed, and the mathematical description and analysis method were given below. Firstly, a new platform configuration is proposed, and then using the kinematic principle to analyze the constraint mode of the platform, we determined the degree of freedom of the platform and came to the platform attitude through analysis.

### II. KINEMATIC PRINCIPLE

Because of the basic position of the constraint, freedom and position in kinematics, we can know that the kinematic method is essentially the analysis of the constraints, freedom and pose of the mechanism, here is a brief introduction.

Constraints can be divided into point contact constraint and kinematic chain constraint. Line contact and surface contact can be regarded as a form of point contact since point contact is one of the most basic constraints in mechanical structure. As shown in Figure 1, the object is subject to a single point of contact and multiple points of contact, object B is represented by the form of a matrix:

$$C = (W_1 \quad W_2 \quad \dots \quad W_n) = \begin{bmatrix} w_1 & w_2 & \dots & w_n \\ r_1 \times w_1 & r_2 \times w_2 & \dots & r_n \times w_n \end{bmatrix} \text{ formula (1)}$$

Matrix C is called the constraint matrix,  $W_1 \quad W_2 \quad W_3 \dots W_n$  is the representation of point contact constraint,  $w_1 \quad w_2 \quad w_3 \dots w_n$  is the direction vector of the contact force,  $r_1 \quad r_2 \quad r_3 \dots r_n$  is the position vector of the contact force.

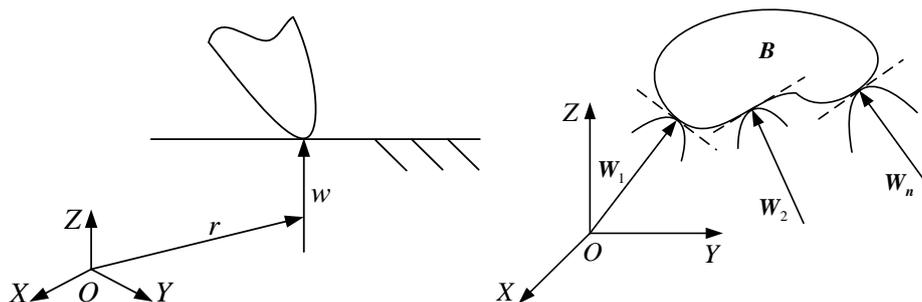


Fig.1a Single point contact Fig.1b Multi point contact

The degree of freedom represents the possible movement of an object in a certain which means that it can be said by virtual displacement. Virtual displacement is the possible displacement of an object in a certain. According to the categories of spinors (pitch of the situation), we can define the object of three degrees of freedom: the rotation degree of freedom, translational degrees of freedom and spiral degrees of freedom<sup>[9-10]</sup>.

As the pitch of the rotation  $p_t = 0$ , Rotational degrees of freedom can be expressed as:

$$\delta T = \delta \rho T_0 = \delta \rho [t \quad l \times t] \text{ formula (2)}$$

Wherein  $\delta \rho$  is object rotation displacement;  $T_0$  is unit screw;  $t$  is the direction vector of  $T_0$ ;  $l$  is the position vector of  $T_0$ ;  $p_t$  is the pitch of  $T_0$ .

Pose position and orientation are the position and pose of the object. A fixed coordinate system is selected and a coordinate system is fixed on the object. The so-called position is the three coordinates of the origin of the fixed coordinate system; The attitude is the direction of the three axis in the fixed coordinate system. The pose of an object is the position and attitude of the two coordinate systems.

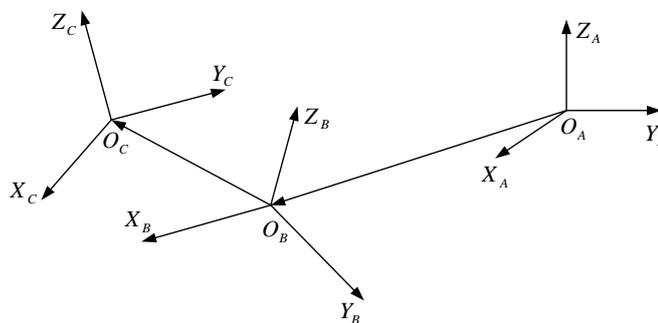


Fig.2 Transform relationship

As shown in Figure 2, the transformation of the object in the position and pose of the object in different coordinates. Transformation equation for the relative coordinate system of  $\{C\}$  in  $\{A\}$ :

$$T_{AC} = T_{AB} T_{BC} \text{ formula (3)}$$

Wherein  $T_{AC}$  is the position and orientation matrix of coordinate system  $O_C X_C Y_C Z_C$  with respect to  $O_A X_A Y_A Z_A$ ;  $T_{AB}$  is the position and orientation matrix of coordinate system  $O_B X_B Y_B Z_B$  with respect to  $O_A X_A Y_A Z_A$ ;  $T_{BC}$  is the position and orientation matrix of coordinate system  $O_C X_C Y_C Z_C$  with respect to  $O_B X_B Y_B Z_B$ .

### III. TWO DOF SPHERICAL PARALLEL PLATFORM

#### 3.1 Platform configuration

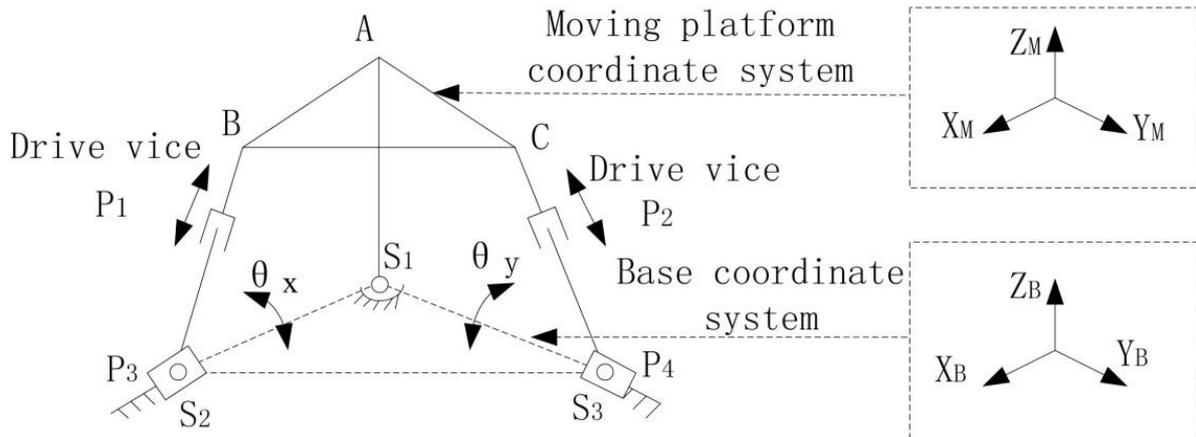


Fig.3 Platform configuration and coordinate system

As shown in Figure 3: 2-PSP&1-S two degree of freedom spherical parallel platform. The platform mainly includes the dynamic platform, the base, a spherical S1 and Kinematic chain P1S2 and P2S3 which composed of two moving pairs and spherical pairs. Spherical S1 is the fulcrum of the movement of the platform. Mobile P1 and P2 are the drivers of vice. The platform can be used for spherical motion around the center of S1 under the driving effect.

#### 3.2 Platform degree of freedom

As shown in Figure 3, the moving platform coordinate system and the moving platform  $O_M X_M Y_M Z_M$  are fixed, and the coordinate origin is selected at the A point. Coordinate axes  $O_M X_M$  and  $O_M Y_M$  are parallel to the AB and AC of the moving platform.  $O_M Z_M$  is determined by the right hand rule. The base coordinate system of the base is 1 and the base is fixedly connected. The origin of the coordinate selection in the center of  $S_1$ ,  $O_B X_B$  and  $O_B Y_B$  are parallel to the  $S_1 S_2$  and  $S_1 S_3$  of the base.  $O_B Z_B$  is determined by the right hand rule.

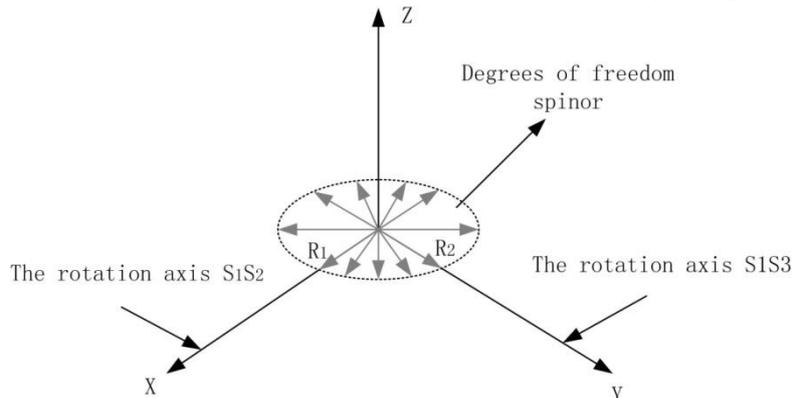


Fig.4 Geometric description of the degree of freedom of the platform

The platform can be defined with three input states: 0- state, 1- state and 2- state. 0- status refers to the natural placement of the platform, the six degrees of freedom on the platform are bound by six points, platform has a certain position and attitude; In the 1- state, the platform motion chain P1S2 or P2S3 is used as the input link, The degree of freedom of the platform is one; In the 2- state, the system is described by a geometric description of the degree of freedom of the system by means of two kinematic chains, which is shown in Figure 4. Based on the structure and motion characteristics of platform acquired by DOF pattern analysis, it can be sure that  $S_1 S_2$  and  $S_1 S_3$  are the axis of rotation of the platform. Therefore, it is found the two rotation degrees of freedom about the X and Y axes unrestricted.

### 3.3 Platform attitude

Set  $R_x(\alpha)$  and  $R_y(\beta)$ , respectively, as a platform to rotate around the matrix X-axis and Y-axis, The attitude matrix platform can be expressed as:

$$T = R_y(\beta)R_x(\alpha) \text{ formula (4)}$$

Wherein  $R_x(\alpha)$  and  $R_y(\beta)$  can be expressed as:

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \text{ formula (5)}$$

Where  $\alpha, \beta$  is Euler angles.

## IV. CONCLUSION

(1) This paper proposes a simple structure, novel two DOF spherical parallel platforms, it can be extended to microelectronics manufacturing, laser engineering and other fields;

(2) The concept of position and attitude is introduced into kinematic design, kinematic design method can be more widely used to deal with the problem of the movement of the mechanism, so as to expand the application range of kinematic design;

(3) On the platform kinematic analysis, it can provide a reference for the development of the follow-up attitude positioning algorithm

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